Super Absorption Boundary Condition for Guided Waves in the 3-D TLM Simulation

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Abstract—**A super absorption boundary condition (SABC) for the 3-D symmetrical condensed node transmission line matrix (3-D SCN TLM) method is presented. As both an electric field and a magnetic field exist at the same time and position, SABC may be more suitable for the 3-D SCN TLM method than for the finite difference time domain (FD-TD) method. To illustrate the effectiveness of the boundary condition, SABC is used to truncate the computational domain of an open microstrip line. It performs better than the Higdon first- and second-order conditions and the Higdon second-order condition with spurious mode suppressed.**

I. Introduction

MANY studies have been carried out to represent the open boundary condition in the FD-TD and TLM methods. Mur's absorbing boundary condition (Mur ABC) [1] is widely used among FD-TD methods. Higdon's one-way equation absorbing boundary condition (Higdon ABC) [2] is easy to apply to waveguide structures with an inhomogeneous cross-section because the only derivatives to be calculated are those normal to the boundary. The detailed implementation of the Higdon ABC for the 3-D SCN TLM method was reported in [3].

Another approach for optimizing the boundary condition is the SABC, which was first proposed for the FD-TD method [4]. This condition can be applied to several absorbing methods and it dramatically reduces the numerical error caused by the boundary reflection, even though first-order boundary conditions are used. The SABC may be more suitable for the 3-D SCN TLM method than for the FD-TD method because both an electric field and magnetic field exist at the same time and position in the 3-D SCN TLM method.

This letter presents the SABC implementation of the 3-D SCN TLM method and shows that it achieves significant improvement on Higdon first-order ABC by using this technique.

II. Super Absorption Boundary Condition

The super absorption boundary condition uses both the electric field and the magnetic field, instead of just one or the other, to reduce the total error of a mismatched boundary. The concept of this technique is as follows. If the electromagnetic wave collides with a discontinuity, a reflection wave is generated that goes backward. In other words, either its electric field or magnetic field must have the opposite sign

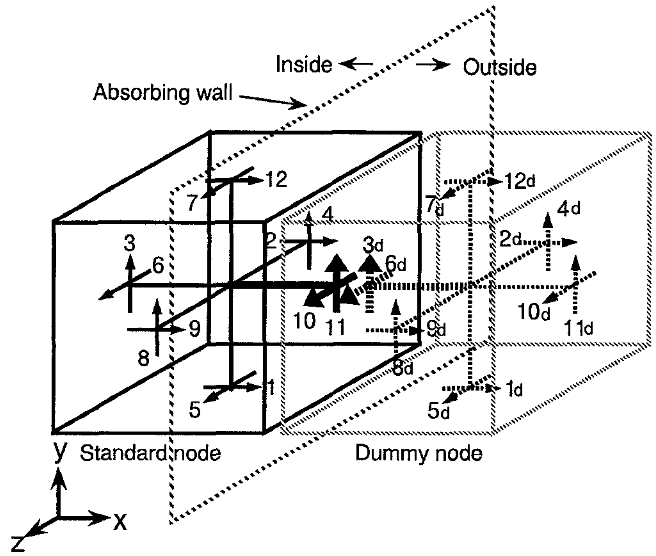

Fig. 1. 3D-TLM symmetrical condensed node with absorbing boundary.

to the input wave. One can use this feature to significantly reduce the reflection error compared to the one in the original boundary condition.

Fig. 1 shows the boundary surface where the super absorption technique is applied. We assume a "dummy" node that is outside the absorbing wall. Between the branches out of the standard node and the dummy one, the following relation is satisfied at the interface, as shown in [5]:

$$\begin{aligned} V_{11}^i(x, t+\Delta t) &= V_y\left(x+\frac{\Delta x}{2}, t-\frac{\Delta t}{2}\right) - ZI_x\left(x+\frac{\Delta x}{2}, t+\frac{\Delta t}{2}\right) \\ &= V_{03d}^r(x+\Delta x, t) \qquad (1) \end{aligned}$$

$$\begin{aligned} V_{11}^r(x, t) &= V_y\left(x+\frac{\Delta x}{2}, t+\frac{\Delta t}{2}\right) + ZI_x\left(x+\frac{\Delta x}{2}, t+\frac{\Delta t}{2}\right) \\ &= V_{03d}^i(x+\Delta x, t+\Delta t). \qquad (2) \end{aligned}$$

Here, if $t + \Delta t/2$ is assumed to be a present time step, V_{11}^i and V_{11}^r are the input and reflected impulses in the next and previous time steps and V_{03d}^i and V_{03d}^r are the input and reflected impulses in the previous and next time steps. V_y and I_x are the total voltage and current on link line 3 and they are equivalent to the electric field E_y and magnetic field H_z. Z is the characteristic impedance of the link line. The V_{11}^i in (1)

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can be obtained if the V_y and I_x are determined. The goal in this problem is then to make the best possible approximation of the V_y and I_x. As the left side of (2) is the previous time step value, it can be regarded as a true value obtained in the standard TLM scattering process. Then

$$V_y^{(e)} + ZI_x^{(e)} = V_{11}^r \tag{3}$$

$$V_y^{(c1)} + ZI_x^{(c1)} = V_{11}^r \tag{4}$$

where the symbol (e) would be the solution if no boundary were present at the plane and the symbol $(c1)$ denotes the value calculated using an initial ABC for V_y. The difference between $V_y^{(e)}$ and $V_y^{(c1)}$ is the error caused by the mismatched boundary condition. Therefore, the error can be written as

$$errorE = V_y^{(e)} - V_y^{(c1)} = Z(I_x^{(e)} - I_x^{(c1)}). \tag{5}$$

Let the same ABC for V_y be applied to I_x. The error is

$$errorH = I_x^{(e)} - I_x^{(c2)}. \tag{6}$$

The expression $I_x^{(c2)}$ is the I_x calculated by the ABC. Here, since we applied the same ABC to V_y and I_x, the relationship between the errors may be the same as that between the electric field and the magnetic field

$$errorE = -errorZerrorH \tag{7}$$

where $errorE$ is assumed to be $Z/\sqrt{\varepsilon_{\text{eff}}}$. In the FD-TD method [4], we must approximate the time and position difference, but the 3-D SCN TLM method simultaneously defines both the electric and magnetic field at the same position, so no approximation is needed. From (5)–(7), we get

$$I_x^{(e)} = \frac{ZI_x^{(c1)} + errorZI_x^{(c2)}}{Z + errorZ}. \tag{8}$$

Finally, we can obtain the V_{11}^i from (1), (3), and (8) as

$$V_{11}^i = V_y^{(e)} - ZI_x^{(e)} = V_{11}^r - 2ZI_x^{(e)}. \tag{9}$$

V_{10}^i is computed by the same process.

III. Numerical Results

The geometry studied is open microstrip line with conductor width $W = 1.0$ mm and zero thickness. The substrate of the microstrip line has a dielectric constant of 4.0 and a thickness of 0.5 mm. The bottom wall is the electric wall and one side wall is the magnetic wall to represent the symmetry boundary condition. The other side wall ($A = 2.5$ mm), top wall ($B = 2.5$ mm), and the front and far ends (yz-plane) are terminated with several ABC's. The space steps in the x, y, and z directions are set to be of equal length, and $\Delta x = \Delta y = \Delta z = \Delta l = 0.125$ mm. The time step $\Delta t = 0.5\Delta l/c$, where c is the speed of light in air. The line is excited with a Gaussian pulse of width $50\Delta t$. The first-order Higdon ABC [3] is used for the initial ABC to apply the SABC.

Fig. 2 compares the current errors caused by four different boundary conditions: 1) first-order Higdon ABC ($\alpha_x = 0.0, \alpha_y = \alpha_z = 0.05/\Delta l, \varepsilon_{rx,\text{eff}} = \varepsilon_{rz,\text{eff}} = 3.1, \varepsilon_{ry,\text{eff}} = 1.0$); 2) section-order Higdon ABC ($\alpha_{x1} = \alpha_{x2} = 0.0, \alpha_{y1} - \alpha_{z1} = 0.05/\Delta l, \alpha_{y2} = \alpha_{z2} = 0.06/\Delta l, \varepsilon_{rx,\text{eff1}} = \varepsilon_{rz,\text{eff1}} = 3.1, \varepsilon_{ry,\text{eff1}} = 1.0, \varepsilon_{rx,\text{eff2}} = \varepsilon_{rz,\text{eff2}} = 3.6, \varepsilon_{ry,\text{eff2}} = 1.0$); 3) second-order Higdon ABC with the spurious mode suppressed; and 4) first-order Higdon ABC with super absoprtion boundary condition applied. Here, α is the attenuating constant and ε_r is the relative dielectric constant. These errors are found by simulating the microstrip with a computational domain of $50\Delta x \times 20\Delta y \times 20\Delta z$ cells. The numerical solution, used as the reference data to find reflection errors due to several ABCs, is computed with a computational domain of $150\Delta x \times 40\Delta y \times 40\Delta z$ cells and terminated with the SABC. In this case, the reflection wave from the far end in the x-direction has not appeared yet. Fig. 2 shows the SABC applied to the first-order Higdon provides better accuracy than the first- and second-order Higdon conditions, and the same accuracy as the second-order Higdon condition with spurious mode suppressed [5]. Fig. 3 compares the characteristic impedances of several ABC's, and it can be seen that the accuracy of the computed characteristic impedance is very sensitive to the error in the ABC. In this figure, the second-order solution is small in the lower frequency region. This is because the second-order Higdon condition leads to an unacceptable dc offset in the reflected current wave, as shown in Fig. 2. Even though the offset is, to some extent, reduced by considering the spurious

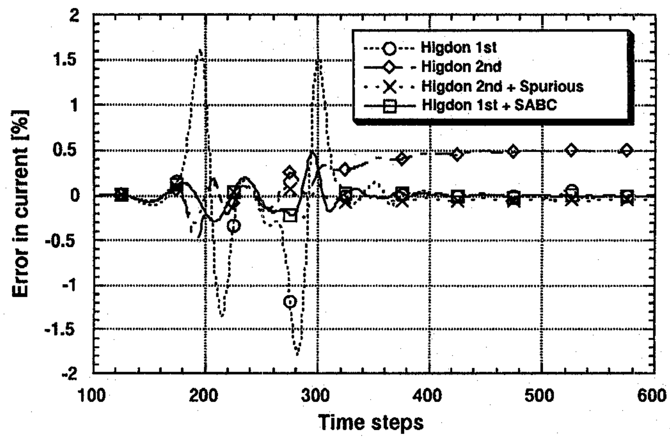

Fig. 2. Comparison of errors in current caused by various ABC's.

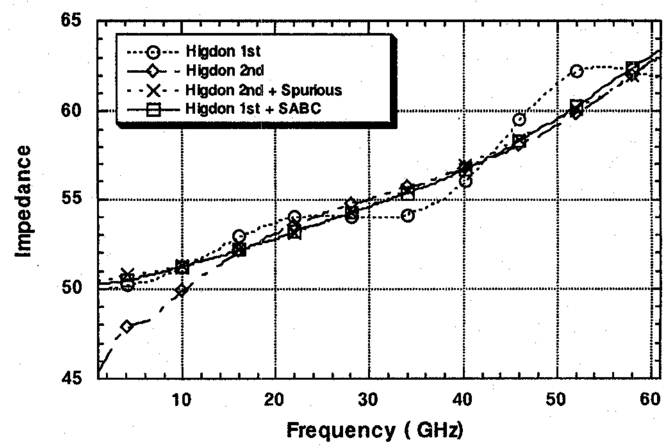

Fig. 3. Comparison of impedances caused by various ABC's.

modes of the 3-D SCN TLM, the super absorption condition applied to the first-order Higdon ABC clearly provides the best performance.

IV. Conclusion

This letter has presented the implementation of the super absorption technique to the 3-D SCN TLM and shown its effectiveness using an open microstrip line structure as an example. The SABC applied to the first-order Higdon ABC produces more accurate and stable results than other conditions. Moreover, as the SABC uses only one previous time step value and one inner space step value, it does not need as much computer memory as the higher-order derivative conditions.

References

[1] G. Mur, "Absorbing boundary conditions for the finite-difference approximation of the time-domain electromagnetic fields equations," *IEEE Trans. Electromagn. Compat.*, vol. EMC-23, pp. 377–382, Nov. 1981.

[2] R. L. Higdon, "Numerical absorbing boundary condition for the wave equation," *Math. Computation*, vol. 49, no. 179, pp. 65–91, July 1987.

[3] C. Eswarappa and W. J. R. Hoefer, "One-way equation absorbing boundary conditions for 3-D TLM analysis of planar and quasi-planar structures," *IEEE Trans. Microwave Theory Tech.*, vol. 42, no. 9, pp. 1669–1677, Sept. 1994.

[4] K. K. Mei and J. Fang, "Superabsorption—A method to improve absorbing boundary conditions," *IEEE Trans. Antennas Propagat.*, vol. 40, no. 9, pp. 1001–1010, Sept. 1992.

[5] Z. Chen, M. M. Ney, and W. J. R. Hoefer, "Absorbing and connecting boundary conditions for the TLM method," *IEEE Trans. Microwave Theory Tech.*, vol. 41, no. 11, pp. 2016–2024, Nov. 1993.